No Radial Excitations in Low Energy QCD. I.

Diquarks and Classification of Mesons

Tamar Friedmann*

Massachusetts Institute of Technology

Cambridge, MA 02139, USA

Abstract

We propose a new schematic model for mesons in which the building blocks are quarks and flavor-antisymmetric diquarks. The outcome is a new classification of the entire meson spectrum into quark-antiquark and diquark-antidiquark states which reveals that there are no radially excited hadrons: all mesons which have so far been believed to be radially excited are orbitally excited diquark–antidiquark states; similarly, there are no radially excited baryons. The classification also leads to the introduction of *isorons* (iso-hadrons), which are analogs of atomic isotopes, and their *magic quantum numbers*, which are analogs of the magic numbers of the nuclear shell model. The magic quantum numbers of isorons match the quantum numbers expected for low-lying glueballs in lattice QCD. We observe that interquark forces in mesons behave substantially differently from those in baryons: qualitatively, they are color–magnetic in mesons but color–electrostatic in baryons. We comment on potential models and the hydrogen atom. The implications of our results for confinement and asymptotic freedom are discussed in our companion paper [1].

*E-mail: tamarf at mit.edu

1 Introduction

The quark model [2] has been for many years the accepted framework for classifying the hadron spectrum. According to this model, quarks are the building blocks for all the hadrons: mesons are bound states of a quark and an antiquark ($q\bar{q}$) and baryons are bound states of three quarks (qqq).

However, not only quarks may be building blocks of hadrons. Diquarks, which are bound configurations of two quarks, may also be building blocks. This approach, originally explored as early as the 1960's (for reviews see [3, 4, 6]), has been revisited following a surge of experimental and theoretical interest in pentaquarks ($qqqq\bar{q}$)[5, 6, 7].[1]

In [9, 10], diquarks were used as building blocks in a systematic classification of all known baryons. As to mesons, while some mesons have been viewed as having diquarks as constituents – to name just two examples, the light scalar mesons were interpreted as diquark-antidiquark states [11], as were several charmed mesons [12, 13] – diquarks have never been used systematically as building blocks for the classification of *all* known mesons.

We set out to undertake this job. Our purpose is to find out whether the entire meson spectrum can be re–classified with the aid of diquarks, and whether we can learn anything new about QCD in the process.

We construct a model in which certain diquark configurations, selected for us by the flavor structure of meson phenomenology, are building blocks for mesons in addition to, and on equal footing with, the quarks of the traditional quark model. These diquarks are the two flavor-antisymmetric ones. One of the two coincides with the most well–known "good" diquark which is antisymmetric in all quantum numbers; the other has been previously unfairly neglected.

Our model reclassifies the meson spectrum into quark-antiquark and diquark-antidiquark states and reassigns L and S quantum numbers to the mesons. Thus, diquark-antidiquark states are no longer "exotic" as they used to be named, but they are naturally integrated into the model.

[1]Experiments eventually showed that the pentaquark Θ^+ does not exist [8]; as Robert Jaffe said (Harvard seminar, 2004), "pentaquarks might come and go, but the diquarks are here to stay."

In the process, a new notion of *isorons* (iso-hadrons) comes inevitably into the picture, along with their *magic J^{PC} quantum numbers*. The isorons are the natural analogs of isotopes or isotones in atomic or nuclear physics, and their magic J^{PC} quantum numbers are analogous to the magic numbers of the nuclear shell model. In the nuclear shell model, it was spin-orbit couplings which was the magic behind the magic numbers. Here, it remains an open problem to understand what is behind the magic J^{PC} of isorons. It is striking that the magic J^{PC} of isorons match the quantum numbers predicted for low-lying glueballs by lattice QCD.

Most significantly, we find that there are no radially excited mesons: mesons that have been believed to be radially excited quark–antiquark states are orbitally excited diquark–antidiquark states. We also find that the same is true for baryons: the baryons that have so far been considered to be radially excited appear to be an orbitally excited configuration of two diquarks and an antiquark.

All in all, *there are no radial excitations in the hadron spectrum*. Therefore, any model which allows for radial excitations in the hadron spectrum cannot be a model of low–energy QCD.

While our original motivation for this work was the study of the role of diquarks in mesons, the results of this study embrace the entire hadron spectrum. In fact, our results have implications regarding the dynamics of the strong force, confinement, and beyond. Those implications are treated separately in our companion paper, "No Radial Excitations in Low Energy QCD. II. The Shrinking Radius of Hadrons," [1].

2 Extended Schematic Model for Mesons

2.1 A few good diquarks

The first question we are faced with when constructing our model is which diquark configurations really are the building blocks for mesons, in addition to and on equal footing with the quarks of the quark model.

This question would be easy to answer if the interquark forces of low–energy QCD were known; if that were the case, we would know which diquark configurations are attractive and those would be our building blocks. Since this is not the case, we instead derive the diquark building blocks from the following aspect of meson phenomenology: in the light meson sector, where the flavor group is $SU(3)$, all observed meson multiplets are flavor nonets. Therefore, the diquarks must be those for which diquark-antidiquark configurations would form only flavor nonets.

To figure out which diquarks satisfy this requirement, we first note that since light quarks are in the $\mathbf{3}_f$ flavor representation, light diquarks may form either flavor sextets or antitriplets:

$$\mathcal{Q} = qq: \quad \mathbf{3}_f \otimes \mathbf{3}_f = \mathbf{6}_f \oplus \bar{\mathbf{3}}_f \qquad\qquad SU(3)_f, \tag{1}$$

where $\mathcal{Q}$ stands for a diquark. The sextet $\mathbf{6}_f$ is symmetric under flavor exchange of the two quarks, while the antitriplet $\bar{\mathbf{3}}_f$ is antisymmetric under this exchange. Now, the only combination of a diquark and an antidiquark that forms exactly a flavor nonet and no larger multiplet is one in which the diquark is a flavor antitriplet and the antidiquark is a flavor triplet. This combination leads to the representations

$$\mathcal{Q}\bar{\mathcal{Q}}: \quad \bar{\mathbf{3}}_f \otimes \mathbf{3}_f = \mathbf{8}_f \oplus \mathbf{1}_f \qquad\qquad SU(3)_f, \tag{2}$$

which are nonets; the flavor sextet representation of the diquark would have led to flavor multiplets larger than nonets, which as noted above are not observed.

Therefore, our building-block diquarks must be those that are in an antisymmetric configuration under flavor exchange. It is now natural to take this flavor-antisymmetry to be the case not just in the light meson sector but also when we include heavy flavors. For $SU(4)$ flavor, which includes the charm quark, the diquarks form the representations

$$\mathcal{Q} = qq: \quad \mathbf{4}_f \otimes \mathbf{4}_f = \mathbf{10}_f \oplus \bar{\mathbf{6}}_f \qquad\qquad SU(4)_f, \tag{3}$$

where the $\mathbf{10}_f$ is symmetric and the $\bar{\mathbf{6}}_f$ is antisymmetric under flavor exchange. The diquark building blocks live in the antisymmetric $\bar{\mathbf{6}}_f$.

Now we list in Table 1 (see also [14]) the flavor, spin, and color states of all the totally antisymmetric configurations of two quarks which are in their lowest orbital. For any given

Table 1: Diquark configurations (adopted from Jaffe [14], and adding the $SU(4)_f$ and $\mathcal{H}_{CE}$ columns). "A" and "S" stand for "Antisymmetric" and "Symmetric" representations, respectively. For a discussion of $\mathcal{H}_{CM}$ and $\mathcal{H}_{CE}$, see Section 5.

	Flavor		*Spin*	*Color*	$\mathcal{H}_{CM}$	$\mathcal{H}_{CE}$
	$SU(3)_f$	$SU(4)_f$	$SU(2)_s$	$SU(3)_c$		
$\mathcal{Q}_1$	$\bar{\mathbf{3}}_f(A)$	$\bar{\mathbf{6}}_f(A)$	$\mathbf{1}_s(A)$	$\bar{\mathbf{3}}_c(A)$	-8	$-8/3$
$\mathcal{Q}_2$	$\bar{\mathbf{3}}_f(A)$	$\bar{\mathbf{6}}_f(A)$	$\mathbf{3}_s(S)$	$\mathbf{6}_c(S)$	$-4/3$	$4/3$
$\mathcal{Q}_3$	$\mathbf{6}_f(S)$	$\mathbf{10}_f(S)$	$\mathbf{3}_s(S)$	$\bar{\mathbf{3}}_c(A)$	$8/3$	$-8/3$
$\mathcal{Q}_4$	$\mathbf{6}_f(S)$	$\mathbf{10}_f(S)$	$\mathbf{1}_s(A)$	$\mathbf{6}_c(S)$	4	$4/3$

row in the table, the product of all three states must be antisymmetric. There are four such configurations, named $\mathcal{Q}_1$, $\mathcal{Q}_2$, $\mathcal{Q}_3$, and $\mathcal{Q}_4$. The first two ($\mathcal{Q}_1$ and $\mathcal{Q}_2$) are the flavor–antisymmetric diquark configurations which are the building blocks for mesons. The $\mathcal{Q}_1$ is also antisymmetric under spin and color; it happens to be the one that played a central role as a proposed constituent of pentaquarks in [7] and has come to be known as the "good" diquark. The other flavor-antisymmetric diquark, $\mathcal{Q}_2$, is symmetric under spin and color, and seems to have been unfairly neglected. We will discuss $\mathcal{Q}_3$ in Section 4.1 as a building block for baryons; $\mathcal{Q}_4$ does not qualify as a building block for any baryon.

2.2 Meson quantum numbers

We now have three types of building blocks for mesons: the quarks q and the two flavor–antisymmetric diquarks $\mathcal{Q}_1$ and $\mathcal{Q}_2$. Armed with these, we now work out the meson quantum numbers that we can expect to arise. See Tables 2a, 2b, and 2c.

Color

Mesons, like all hadrons, must be color singlets. There are three combinations of our building blocks that yield such objects:

Tables 2a, 2b, 2c: Meson quantum numbers for $q\bar{q}$, $\mathcal{Q}_1\bar{\mathcal{Q}}_1$, and $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ up to $L = 3$. The third column is derived using equations (12) and (13). A $\surd$ indicates that at least one member of the corresponding light nonet has been observed (see Tables 3a, 3b); a dot "$\bullet$" indicates that this nonet consists mainly of well-established mesons.

Table 2a: $q\bar{q}$

L	S	J^{PC}	$^{2S+1}L_J$	
0	0	0^{-+}	$^1S_0$	$\surd\bullet$
0	1	1^{--}	$^3S_1$	$\surd\bullet$
1	0	1^{+-}	$^1P_1$	$\surd\bullet$
1	1	2^{++}	$^3P_2$	$\surd\bullet$
		1^{++}	$^3P_1$	$\surd\bullet$
		0^{++}	$^3P_0$	$\surd\bullet$
2	0	2^{-+}	$^1D_2$	$\surd\bullet$
2	1	3^{--}	$^3D_3$	$\surd\bullet$
		2^{--}	$^3D_2$	$\surd$
		1^{--}	$^3D_1$	$\surd\bullet$
3	0	3^{+-}	$^1F_3$	$\surd$
3	1	4^{++}	$^3F_4$	$\surd\bullet$
		3^{++}	$^3F_3$	
		2^{++}	$^3F_2$	$\surd\bullet$

Table 2b: $\mathcal{Q}_1\bar{\mathcal{Q}}_1$

L	S	J^{PC}	$^{2S+1}L_J$	
0	0	0^{++}	$^1S_0$	$\surd\bullet$
1	0	1^{--}	$^1P_1$	$\surd\bullet$
2	0	2^{++}	$^1D_2$	$\surd$
3	0	3^{--}	$^1F_3$	$\surd$

Table 2c: $\mathcal{Q}_2\bar{\mathcal{Q}}_2$

L	S	J^{PC}	$^{2S+1}L_J$	
0	0	0^{++}	$^1S_0$	
0	1	1^{+-}	$^3S_1$	
0	2	2^{++}	$^3S_2$	
1	0	1^{--}	$^1P_1$	
1	1	2^{-+}	$^3P_2$	$\surd\bullet$
		1^{-+}	$^3P_1$	$\surd$
		0^{-+}	$^3P_0$	$\surd\bullet$
1	2	3^{--}	$^5P_3$	
		2^{--}	$^5P_2$	$\surd\bullet$
		1^{--}	$^5P_1$	$\surd$
2	0	2^{++}	$^1D_2$	$\surd$
2	1	3^{+-}	$^3D_3$	
		2^{+-}	$^3D_2$	
		1^{+-}	$^3D_1$	$\surd$
2	2	4^{++}	$^5D_4$	
		3^{++}	$^5D_3$	
		2^{++}	$^5D_2$	$\surd$
		1^{++}	$^5D_1$	$\surd$
		0^{++}	$^5D_0$	$\surd$
3	0	3^{--}	$^1F_3$	$\surd$
3	1	4^{-+}	$^3F_4$	$\surd$
		3^{-+}	$^3F_3$	
		2^{-+}	$^3F_2$	$\surd$
3	2	5^{--}	$^5F_5$	
		4^{--}	$^5F_4$	
		3^{--}	$^5F_3$	
		2^{--}	$^5F_2$	
		1^{--}	$^5F_1$	

$$q\bar{q} \;:\quad \mathbf{3}_c \otimes \bar{\mathbf{3}}_c = \mathbf{8}_c \oplus \mathbf{1}_c \qquad SU(3)_c, \tag{4}$$

$$\mathcal{Q}_1\bar{\mathcal{Q}}_1 \;:\quad \bar{\mathbf{3}}_c \otimes \mathbf{3}_c = \mathbf{8}_c \oplus \mathbf{1}_c \qquad SU(3)_c, \tag{5}$$

$$\mathcal{Q}_2\bar{\mathcal{Q}}_2 \;:\quad \mathbf{6}_c \otimes \bar{\mathbf{6}}_c = \mathbf{27}_c \oplus \mathbf{8}_c \oplus \mathbf{1}_c \qquad SU(3)_c, \tag{6}$$

so we have three types of mesons corresponding to the three appearances of $\mathbf{1}_c$.

Flavor

As we ensured in Section 2.1, in the light quark sector with an $SU(3)$ flavor group all our mesons – $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ and $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ as well as $q\bar{q}$ – live in flavor nonets:

$$q\bar{q} \;:\quad \mathbf{3}_f \otimes \bar{\mathbf{3}}_f = \mathbf{8}_f \oplus \mathbf{1}_f \qquad SU(3)_f, \tag{7}$$

$$\mathcal{Q}_i\bar{\mathcal{Q}}_i \;:\quad \bar{\mathbf{3}}_f \otimes \mathbf{3}_f = \mathbf{8}_f \oplus \mathbf{1}_f \qquad SU(3)_f, \tag{8}$$

where $\mathcal{Q}_i\bar{\mathcal{Q}}_i$ denotes both $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ and $\mathcal{Q}_2\bar{\mathcal{Q}}_2$.

When we include the charm quark, the flavor group is $SU(4)$, and the $q\bar{q}$ lives in

$$q\bar{q} \;:\quad \mathbf{4}_f \otimes \bar{\mathbf{4}}_f = \mathbf{15}_f \oplus \mathbf{1}_f \qquad SU(4)_f, \tag{9}$$

while $\mathcal{Q}_i\bar{\mathcal{Q}}_i$ live in

$$\mathcal{Q}_i\bar{\mathcal{Q}}_i \;:\quad \bar{\mathbf{6}}_f \otimes \mathbf{6}_f = \mathbf{20}_f \oplus \mathbf{15}_f \oplus \mathbf{1}_f \qquad SU(4)_f. \tag{10}$$

These are the flavor multiplets of our mesons.

Spin, parity, and charge

The total spin J, parity P, and charge C quantum numbers are denoted J^{PC}. The total spin J is given as usual by adding orbital (L) and internal spin (S) angular momenta:

$$J = L \otimes S\ . \tag{11}$$

The parity and charge quantum numbers are given by:

$$q\bar{q} \;:\quad P = (-1)^{L+1}\ , \qquad C = (-1)^{L+S} \tag{12}$$

$$\mathcal{Q}_i\bar{\mathcal{Q}}_i \;:\quad P = (-1)^{L}\ , \qquad C = (-1)^{L+S}\ . \tag{13}$$

One should note that the orbital angular momentum L is between the quark and antiquark or diquark and antidiquark; the internal orbital angular momentum of our building blocks is zero.

We list all the allowed J^{PC} quantum numbers along with their corresponding L and S for $L \leq 3$ in Tables 2a ($q\bar{q}$), 2b ($\mathcal{Q}_1\bar{\mathcal{Q}}_1$), and 2c ($\mathcal{Q}_2\bar{\mathcal{Q}}_2$).

3 Re–classification of Mesons

We are now ready to re-classify the meson spectrum. We carry out the following procedure: we compile a list – from the Particle Listings in the PDG [8] – of all the mesons[2] along with their masses, flavor and J^{PC} quantum numbers. We arrange them into flavor multiplets with approximately degenerate masses and common J^{PC}; the light mesons form either full or partial flavor nonets. Then, we turn to our tables of meson quantum numbers (Tables 2a, 2b, 2c) for all the occurrences of each J^{PC} and assign each multiplet (or partial multiplet) to an appropriate meson type ($q\bar{q}$, $\mathcal{Q}_1\bar{\mathcal{Q}}_1$, or $\mathcal{Q}_2\bar{\mathcal{Q}}_2$) with specified L and S quantum numbers.

In making the assignment, we make a rough but standard assumption [14, 12, 15, 16], which we call the *"orbital excitation rule,"* that every unit of orbital angular momentum L contributes about .5GeV to the mass of a light meson. Therefore, roughly speaking we expect the following mass ranges for light mesons:

$$\textit{orbital excitation rule:} \qquad \begin{array}{ll} \text{S–waves} & m \leq 1GeV \\ \text{P–waves} & 1 < m < 1.5GeV \\ \text{D–waves} & 1.5 < m < 2GeV \\ \text{F–waves} & 2 < m < 2.5GeV \ , \end{array} \tag{14}$$

and so on.

[2]We omit those listed under "further states" in the PDG, as they have not been confirmed.

This procedure produces the following tables: Table 3a (light mesons), Table 3b (charmed and bottom mesons), and Table 3c (isorons and magic J^{PC}, both light and heavy, to be defined below).[3]

We now analyze the outcome.

3.1 Isorons, magic numbers, and glueballs

In most cases, our procedure above resulted in a unique assignment for each meson, which appears in Tables 3a and 3b. As we were carrying out the procedure, we noticed that sometimes, multiple mesons which carry the same quantum numbers but different masses vied for one available space in the tables. One of these, usually the one most closely degenerate in mass with the relevant multiplet, was placed in that available space. The others are hereby named *isorons*, short for iso-hadrons and analogous to isotopes of atomic physics. Recall the standard definition for isotopes[4]:

> "any of two or more species of atoms of a chemical element with the same atomic number and position in the periodic table and nearly identical chemical behavior but with differing atomic mass or mass number and different physical properties."

Just as with isotopes, we define an isoron to be one of two or more species of mesons with the same quantum numbers but different mass. The isorons are an integral part of the hadronic spectrum, the same way that isotopes are an integral component of the elements. The isorons are listed in Table 3c by J^{PC}.

There are certain J^{PC}'s for which there is an abundance of isorons; we name these "magic J^{PC}" in analogy with the magic numbers of the nuclear shell model [17]. From Table 3c we see that the magic J^{PC} are 0^{-+}, 0^{++}, 2^{++} for light mesons and 1^{--} for heavy mesons.

Strikingly, the magic J^{PC} for light mesons match the J^{PC} expected for low-lying glueballs: lattice QCD calculations indicate that ground state glueballs have $J^{PC} = 0^{++}$ and the first two excited states of glueballs have $J^{PC} = 2^{++}$ and $J^{PC} = 0^{-+}$ [18, 19]. This matching

[3] For a multiplet by multiplet discussion of the process, see Appendix A.

[4] Definition taken from Encyclopaedia Brittanica online.

Table 3a: Our suggested assignments for observed light mesons. Compare with Table 14.2 in the PDG [8]. The second and third columns are the quark/diquark constituents and the orbital and spin quantum numbers, respectively; there is no radial quantum number. A blank space in the fourth column indicates that the appropriate meson has not yet been detected. A dot "•" next to a meson indicates it is considered well–established by the PDG. See Appendix A for a line-by-line discussion. For comparison only, the current PDG assignment (which assumes $q\bar{q}$ constituents and a radial quantum number n), if any, is displayed in the last column.

J^{PC}	constituents	$^{2S+1}L_J$	$I=1$	$I=\frac{1}{2}$	$I=0$		$n^{2S+1}L_J(PDG)$
0^{-+}	$q\bar{q}$	$^1S_0$	•π	•K	•η	•$\eta'(958)$	1^1S_0
0^{-+}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^3P_0$	•$\pi(1300)$	$K(1460)$	•$\eta(1475)$	•$\eta(1295)$	2^1S_0
0^{++}	$\mathcal{Q}_1\bar{\mathcal{Q}}_1$	$^1S_0$	•$a_0(980)$	$\kappa(800)$	•$f_0(980)$	•$f_0(600)$	——
0^{++}	$q\bar{q}$	$^3P_0$	•$a_0(1450)$	•$K_0^*(1430)$	•$f_0(1710)$	•$f_0(1370)$	1^3P_0
0^{++}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^5D_0$		$K_0^*(1950)$	$f_0(2100)$	•$f_0(2020)$	——
1^{--}	$q\bar{q}$	$^3S_1$	•$\rho(770)$	•$K^*(892)$	•$\phi(1020)$	•$\omega(782)$	1^3S_1
1^{--}	$\mathcal{Q}_1\bar{\mathcal{Q}}_1$	$^1P_1$	•$\rho(1450)$	•$K^*(1410)$	•$\phi(1680)$	•$\omega(1420)$	2^3S_1
1^{--}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^5P_1$	$\rho(1570)$				——
1^{--}	$q\bar{q}$	$^3D_1$	•$\rho(1700)$	•$K^*(1680)$	•$\omega(1650)$		1^3D_1
1^{--}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^5F_1$	$\rho(2150)$				——
1^{-+}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^3P_1$	•$\pi_1(1600)$	$K(1630)$			——
1^{++}	$q\bar{q}$	$^3P_1$	•$a_1(1260)$	•$K_1(1400)$	•$f_1(1420)$	•$f_1(1285)$	1^3P_1
1^{++}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^5D_1$	$a_1(1640)$	$K_1(1650)$	$f_1(1510)$		——
1^{+-}	$q\bar{q}$	$^1P_1$	•$b_1(1235)$	•$K_1(1270)$	$h_1(1380)$	•$h_1(1170)$	1^1P_1
1^{+-}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^3D_1$			$h_1(1595)$		——
2^{-+}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^3P_2$	•$\pi_2(1670)$	$K_2(1580)$	$\eta_2(1870)$	•$\eta_2(1645)$	1^1D_2
2^{-+}	$q\bar{q}$	$^1D_2$	•$\pi_2(1880)$				——
2^{-+}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^3F_2$	$\pi_2(2100)$	$K_2(2250)$			——
2^{--}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^5P_2$		•$K_2(1770)$			——
2^{--}	$q\bar{q}$	$^3D_2$		•$K_2(1820)$			1^3D_2
2^{++}	$q\bar{q}$	$^3P_2$	•$a_2(1320)$	•$K_2^*(1430)$	$f_2(1430)$	•$f_2(1270)$	1^3P_2
2^{++}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^1D_2$			•$f_2'(1525)$		——
2^{++}	$\mathcal{Q}_1\bar{\mathcal{Q}}_1$	$^1D_2$	$a_2(1700)$		$f_2(1640)$	$f_2(1565)$	——
2^{++}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^5D_2$			$f_2(1810)$		——
2^{++}	$q\bar{q}$	$^3F_2$		$K_2^*(1980)$	•$f_2(2010)$	•$f_2(1950)$	——*
3^{--}	$q\bar{q}$	$^3D_3$	•$\rho_3(1690)$	•$K_3(1780)$	•$\phi_3(1850)$	•$\omega_3(1670)$	1^3D_3
3^{--}	$\mathcal{Q}_1\bar{\mathcal{Q}}_1$	$^1F_3$	$\rho_3(1990)$				——
3^{--}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^1F_3$	$\rho_3(2250)$				——
3^{+-}	$q\bar{q}$	$^1F_3$		$K_3(2320)$			——
4^{-+}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^3F_4$		$K_4(2500)$			——
4^{++}	$q\bar{q}$	$^3F_4$	•$a_4(2040)$	•$K_4^*(2045)$	$f_4(2220)$	•$f_4(2050)$	1^3F_4
5^{--}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^5F_5$	$\rho_5(2350)$	$K_5^*(2380)$			1^3G_5
6^{++}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^5G_6$	$a_6(2450)$		$f_6(2510)$		1^3H_6

* This nonet was classified as 2^3P_2, a radial excitation, between 1992 and 2002.

Table 3b: Our suggested assignments for observed heavy (charm and bottom) mesons. Compare with Table 14.3 in the PDG [8]. We include the new B_1, B_{s1}, B_2^*, B_{s2}^*. We use the mesons' names as they appear in the PDG for convenience, without agreeing with the radial or orbital assignments that sometimes appear in a meson's name. The second and third columns are the quark/diquark constituents and the orbital and spin quantum numbers, respectively; there is no radial quantum number. A blank space in the fourth column indicates that the appropriate meson has not yet been detected. A dot "•" next to a meson indicates it is considered well–established by the PDG. See Appendix A for a line-by-line discussion. For comparison only, the current PDG assignment (which assumes $q\bar{q}$ constituents and a radial quantum number n), if any, is displayed in the last column.

			Charmed mesons				**Bottom mesons**				$n^{2S+1}L_J$
J^{PC}	constituents	$^{2S+1}L_J$	$I=1^\circ$	$I=\frac{1}{2}$	$I=0$	$I=0$	$I=1^\circ$	$I=\frac{1}{2}$	$I=0$	$I=0$	(PDG)
0^{-+}	$q\bar{q}$	$^1S_0$		$\bullet D$	$\bullet D_s^\sharp$	$\bullet\eta_c(1S)$		$\bullet B^\dagger$	$\bullet B_s^\dagger, B_c^\dagger$	$\eta_b(1S)^\dagger$	1^3S_0
0^{-+}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^3P_0$				$\bullet\eta_c(2S)^\dagger$					2^1S_0
0^{++}	$\mathcal{Q}_1\bar{\mathcal{Q}}_1$	$^1S_0$		$D_0^*(2400)^\sharp$	$\bullet D_{s0}^*(2317)$	$\bullet\chi_{c0}(1P)$				$\bullet\chi_{b0}(1P)$	1^3P_0
0^{++}	$q\bar{q}$	$^3P_0$								$\chi_{b0}(2P)^{\dagger\dagger}$	2^3P_0
1^{--}	$q\bar{q}$	$^3S_1$		$\bullet D^*$	$\bullet D_s^{*\sharp}$	$\bullet J/\psi(1S)$		$\bullet B^{*\dagger}$	$B_s^{*\dagger}$	$\bullet\Upsilon(1S)$	1^3S_1
1^{--}	$\mathcal{Q}_1\bar{\mathcal{Q}}_1$	$^1P_1$				$\bullet\psi(2S)$				$\bullet\Upsilon(2S)$	2^3S_1
1^{--}	$q\bar{q}$	$^3D_1$				$\bullet\psi(3770)$				$\bullet\Upsilon(3S)$	——*
1^{--}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^5F_1$				$\bullet\psi(4040)$				$\bullet\Upsilon(4S)$	——*
1^{++}	$q\bar{q}$	$^3P_1$		$D_1(2420)$	$\bullet D_{s1}(2536)^\sharp$	$\bullet\chi_{c1}(1P)$		$\bullet B_1(5721)^{0\dagger}$	$\bullet B_{s1}(5830)^{0\dagger}$	$\bullet\chi_{b1}(1P)^{\dagger\dagger}$	——**
1^{++}	$\mathcal{Q}_2\bar{\mathcal{Q}}_2$	$^5D_1$			$\bullet D_{s1}(2460)$	$\bullet X(3872)^{\sharp\sharp}$				$\bullet\chi_{b1}(2P)^{\dagger\dagger}$	——**
2^{++}	$q\bar{q}$	$^3P_2$		$\bullet D_2^*(2460)$	$\bullet D_{s2}(2573)^\sharp$	$\bullet\chi_{c2}(1P)$		$\bullet B_2^*(5747)^{0\dagger}$	$\bullet B_{s2}^*(5840)^\dagger$	$\bullet\chi_{b2}(1P)^{\dagger\dagger}$	$1^3P_2^{**}$
2^{++}	$\mathcal{Q}_1\bar{\mathcal{Q}}_1$	$^1D_2$				$\chi_{c2}(2P)$				$\bullet\chi_{b2}(2P)^{\dagger\dagger}$	2^3P_2

$^\circ$ $I=1$ applies only to $\mathcal{Q}_i\bar{\mathcal{Q}}_i$ multiplets; no $I=1$ is expected in charm or bottom $q\bar{q}$ mesons.

† I, J^{PC} need confirmation

†† J needs confirmation.

$^\sharp$ J^P needs confirmation.

$^{\sharp\sharp}$ Quantum numbers not established.

* The $\Upsilon(3S)$ and $\Upsilon(4S)$ are in the PDG but not in Table 14.3. Their names imply that they are believed to be 3^3S_1 and 4^3S_1, respectively. The $\psi(3770)$ is listed in Table 14.3 as 1^3D_1.

** In Table 14.3 of the PDG, the $\chi_{c1}(1P)$ is listed as 1^3P_1, while the $D_1(2420)$ and $D_{s1}(2536)$ are listed as $J^{PC}=1^{+-}$ with 1^1P_1. The $D_{s1}(2460)$ is listed as a 1^{++} with 1^3P_1 and the $\chi_{b1}(2P)$ is listed as 2^3P_1. The B_1, B_{s1}, B_2^*, and B_{s2}^* are in the PDG but not in Table 14.3.

Table 3c: Isorons and magic J^{PC} (see Section 3.1). The magic J^{PC} are 0^{-+}, 0^{++}, 2^{++} for light isorons and 1^{--} for heavy isorons. A dot "$\bullet$" next to a meson indicates it is considered well–established by the PDG. In the glueball table at the bottom, a column with an "X" indicates that the corresponding J^{PC} quantum number is expected for glueballs by lattice QCD.

Isorons						
	0^{-+}	0^{++}	1^{--}	1^{-+}	2^{++}	4^{++}
Light	$\bullet\eta(1405)$ $\eta(1760)$ $\bullet\pi(1800)$ $K(1830)$ $\eta(2225)$	$\bullet f_0(1500)$ $f_0(2200)$ $f_0(2330)$	$\rho(1900)$	$\bullet\pi_1(1400)$	$f_2(1910)$ $f_2(2150)$ $\bullet f_2(2300)$ $\bullet f_2(2340)$	$f_4(2300)$
Heavy			$\bullet\psi(4160)$ $\bullet X(4260)$ $X(4360)$ $\bullet\psi(4415)$ $\Upsilon(10860)$ $\Upsilon(11020)$			

Glueballs						
	0^{-+}	0^{++}	1^{--}	1^{-+}	2^{++}	4^{++}
	X	X			X	

cannot be a coincidence – there must be a deep underlying reason for it. That reason is beyond the scope of this paper.

3.2 No radials

As we can see, a central result of our classification is that there is no radial quantum number, which indicates that there are no radially excited mesons. The meson multiplets which have been believed[5] to be radially excited $q\bar{q}$ are actually orbitally excited $\mathcal{Q}_i\bar{\mathcal{Q}}_i$:

- the second 0^{-+} nonet, which was classified in the literature and in Table 14.2 of the PDG as a radial excitation with $n^{2S+1}L_J = 2^1S_0$, finds its place here as a $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ with ${}^{2S+1}L_J = {}^3P_0$;
- the second 1^{--} nonet, which was classified in the literature and in Table 14.2 of the PDG as a radial excitation with $n^{2S+1}L_J = 2^3S_1$, finds its place here as a $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ with ${}^{2S+1}L_J = {}^1P_1$.

As we show later on (in Section 4), there is no radial quantum number in the baryon sector either, so put together, there are no radial excitations in the entire hadron spectrum.

But, given that there is no radial quantum number, how is it that for so many years it has been believed that radial excitations of hadrons do exist?

One of the main sources for the concept that hadrons may be radially excited goes back to potential models. According to these models, low–energy QCD is described by a quark–quark potential $V(r)$, where r is the distance between the quarks. The potential in these models has two terms: a short–distance term that is Coulomb-like (i.e., proportional to $-1/r$) and analogous to the interaction between the proton and electron in the hydrogen atom, and a long–distance term $V_{conf}(r)$ that increases with r and – according to the models – describes confinement. (For a review of potential models, see [20].)

In these models, the spectrum for quark–antiquark bound states, i.e. mesons, is obtained by solving the Schrödinger equation with the above potential $V(r)$. As with the hydrogen atom, or as with any central potential in non-relativistic quantum mechanics, the resulting

[5] We take Tables 14.2 and 14.3 of the PDG [8] to be the currently accepted quark model classification.

quantum numbers that describe the spectrum include a principal or radial quantum number n. Hence, potential models automatically allow for, and in fact require, radial quantum numbers and radial excitations. Other studies of QCD have also employed analogies with the hydrogen atom; for a recent example see [21].

In contrast, in the early versions of the PDG [22], starting in the 1960's when the quark model was first proposed, mesons were classified only by spin and orbital quantum numbers:

$$^{2S+1}L_J \ . \tag{15}$$

There was no radial quantum number n. Similarly, early discussions of the quark model did not mention radial excitations or a radial quantum number [23]. The quark model certainly does not call for a radial quantum number. Radial quantum numbers for the hadron spectrum appeared in the PDG for the first time only in 1980 [24]. The atomic notation

$$n^{2S+1}L_J \ , \tag{16}$$

which includes the radial quantum number n, was adopted by the PDG for the hadron spectrum only in 1992 [25]. Interestingly enough, the classification of some mesons as radials in the PDG's from 1992 through 2002 was partially retracted in the subsequent versions (compare Table 13.2 of [26] to Table 14.2 of [27] or [8]): their classification as radials was considered far-fetched [28, 29].

Was there ever any *experimental* evidence for a radial quantum number in hadrons? As of now, the internal radial structure of hadrons has not been experimentally probed: all that has been reported so far is a measurement of the form factors of a few low–lying hadrons, from which their charge radius can be inferred (this has been done for $\pi^{\pm}$, $K^{\pm}$, p, Σ^{-}) [8]. So the radial quantum number that ultimately crept into the quark model classification tables and the PDG was actually an artifact of the models rather than a quantity arising from any measured property of hadrons or quarks.

Furthermore, theoretical predictions about radial excitations in hadrons have been known to encounter difficulties: data involving the masses of the candidates for radial excitations shows that they are often significantly lighter than predicted by the models, and data involving their decay modes often does not favor a radial assignment either [30].

In retrospect, it is actually natural that the quantum numbers of hadrons are different from those of atoms. After all, the hydrogen atom, and the entire atomic system, is inherently different from low–energy QCD even if only because atoms are ionizable whereas low–energy QCD is confining.

We leave a more complete discussion of the implications of the result that there are no radially excited hadrons to our companion paper [1].

3.3 No "exotics" or other outcasts

The traditional quark model allows only for $q\bar{q}$ mesons. The term "exotic meson" refers to those mesons which do not fit into the traditional quark model. While for many years there seemed to be a very small number of exotic mesons – the light scalar mesons were the only ones unexpected by the model – more and more exotic mesons have recently been discovered, including several charmed mesons and a few pions. None of these mesons can be adequately explained within the traditional quark model.

Our model embraces these mesons as legitimate constituents of the hadron spectrum, and they are no longer "exotic." Instead, they are made up of $\mathcal{Q}_i\bar{\mathcal{Q}}_i$. These formerly exotic mesons, along with their classification, include:

- the "cryptoexotic" [6] light scalar nonet with $J^{PC} = 0^{++}$ is a $\mathcal{Q}_1\bar{\mathcal{Q}}_1$, ${}^1S_0$ (see also [11]);
- a manifestly "exotic" meson with $J^{PC} = 1^{-+}$ is a $\mathcal{Q}_2\bar{\mathcal{Q}}_2$, ${}^3P_1$ (see also [31]);
- some newly discovered charmed mesons, (see [13]) including:
  - the $D^*_{sJ}(2317)$ with $J^{PC} = 0^{++}$ is a $\mathcal{Q}_1\bar{\mathcal{Q}}_1$, ${}^1S_0$;
  - the $D_{sJ}(2460)$ with $J^{PC} = 1^{++}$ is a $\mathcal{Q}_2\bar{\mathcal{Q}}_2$, ${}^5D_1$;
  - the $X(3872)$ with $J^{PC} = 1^{++}$ is a $\mathcal{Q}_2\bar{\mathcal{Q}}_2, {}^5D_1$.

There are also numerous other mesons which have been just left out of the classification tables of the traditional quark model – see Appendix B, Table 5 for a complete list of the unclassified mesons. These are also embraced into our model, for example:

- some heavier scalar mesons with $J^{PC} = 0^{++}$ now form a nonet which is classified as $\mathcal{Q}_2\bar{\mathcal{Q}}_2$, ${}^5D_0$;

- some vector mesons with $J^{PC} = 1^{++}$ are now $\mathcal{Q}_2\bar{\mathcal{Q}}_2$, ${}^5D_1$;
- some 2^{++} mesons which are now $\mathcal{Q}_1\bar{\mathcal{Q}}_1$, ${}^1D_2$.

3.4 New particles

Our model implies the existence of new particles. Any blank space in Tables 3a and 3b represents a missing meson. In addition, any row in Tables 2a, 2b, and 2c which does not have a "$\surd$" in the rightmost column represents a missing multiplet.

In the online 2009 partial update of PDG-Live (http://pdg.lbl.gov), there is a list of light "further states," which are "states observed by a single group or states poorly established." We did not use these mesons in our study, but quite a few of the blank spaces in the tables may be filled by these mesons if they are eventually confirmed. For example, the $\omega(1960)$ may partially complete the fifth 1^{--} nonet; the $\rho_2(1940)$ and the $\omega_2(1975)$ may partially complete the second 2^{--} nonet; the $a_2(1990)$ may complete the third 2^{++} nonet; the $\omega_3(1945)$ and $\omega_3(2255)$ may partially complete the second and third 3^{--} nonet, respectively; the $b_3(2025)$, $h_3(2025)$, and $h_3(2275)$ may complete the first 3^{+-} nonet; and $\omega_5(2250)$ may partially complete the 5^{--} nonet.

Other mesons whose detection would support our model are those whose quantum numbers are part of our model but are prohibited in the traditional quark model (these are the "manifestly exotic" quantum numbers). These include $J^{PC} = 1^{-+}, 2^{+-}, 3^{-+}$, etc.; these are manifestly exotic with respect to the quark model but they appear in our model as $\mathcal{Q}_2\bar{\mathcal{Q}}_2$. Some $J^{PC} = 1^{-+}$ mesons (the $\pi_1(1400)$ and $\pi_1(1600)$) have already been established and their existence is evidence already supporting our model. It is interesting that the $J^{PC} = 1^{-+}$ pions were detected relatively recently (in 1997 [32]), and in fact acquired well–established status in the PDG only in 2004; we believe their cousins with $J^{PC} = 2^{+-}, 3^{-+}$ will follow suit.

3.5 Mass hierarchies in light nonets

A strange quark is heavier than an up or down quark. Therefore, in the light meson sector, a strange quark constituent makes a meson heavier. As a result, the mass hierarchy within a $\mathcal{Q}_i\bar{\mathcal{Q}}_i$ nonet is expected to be inverted as compared to the mass hierarchy of a $q\bar{q}$ nonet [11, 14, 33]. That is, in $q\bar{q}$ nonets, the $I = 1/2$ mesons (one strange quark) are heavier than the $I = 1$ mesons (no strange quarks), while in $\mathcal{Q}_i\bar{\mathcal{Q}}_i$ nonets the $I = 1/2$ mesons (one strange quark) are lighter than the $I = 1$ mesons (two strange quarks). This is particularly prominent for the first 0^{++} nonet, whose mass hierarchy is clearly inverted, as was first noted in [11].

The results obtained through our classification are consistent with this expected mass hierarchy in almost all cases. However, it should be noted that sometimes, the actual mass hierarchies cannot be read off from Table 3a. For one, the names of the mesons do not always reflect the meson's mass: generally, a meson's name in the PDG does not get updated when mass measurements are improved, sometimes making it appear as though the mass hierarchy in a nonet is the opposite of what it really is. For example, in the 1^{-+} nonet, classified as a $\mathcal{Q}_2\bar{\mathcal{Q}}_2$, the mass of the $\pi_1(1600)$ is actually 1662 MeV[6], and the mass of the $K(1630)$ is 1629 MeV, so the $K(1630)$ is in fact lighter than the $\pi_1(1600)$, making the mass hierarchy inverted as expected.

Also, there are experimental errors in mass measurements that are significant and could make the mass hierarchy of a nonet uncertain. In our classification, in the second 1^{++} nonet, the mass of the $a_1(1640)$ is actually 1647 ± 22 MeV and the mass of the $K_1(1650)$ is 1650 ± 50 MeV, so it could very well be that the $K_1(1650)$ is lighter than the $a_1(1640)$, consistent with an inverted hierarchy of a $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ nonet. In the fourth 1^{--} nonet, where the $\rho(1700)$ has mass 1720 ± 20 MeV and the $K^*(1680)$ has mass 1717 ± 27 MeV, so it could very well be that the $K^*(1680)$ is heavier than the $\rho(1700)$, consistent with a $q\bar{q}$ nonet. In the second 0^{-+} nonet, the $\pi(1300)$ appears lighter than the $K(1460)$, but the mass of the $\pi(1300)$ is 1300 ± 100 MeV, and the $K(1460)$ seems to have been measured only twice over 25 years ago, once giving the mass 1400 MeV and once giving the mass 1460 MeV. Therefore, it is possible that

[6]This mass was reported as 1596 MeV in earlier editions of the PDG.

the $K(1460)$ would eventually be found to be lighter than the $\pi(1300)$, consistent with our $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ assignment. Another example of this kind is the second 0^{++} nonet.

The third 2^{-+} nonet is the only one that at this time appears to have an unexpected mass hierarchy.

3.6 Binding energies of the diquarks

While many of the expected $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ mesons have been observed, the same is not true of the $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ mesons. This fact alone leads us to believe that the $\mathcal{Q}_2$ is less tightly bound than the $\mathcal{Q}_1$.

We can use our classification to compare the binding energies of the $\mathcal{Q}_1$ and $\mathcal{Q}_2$ diquarks because in the light $J^{PC} = 3^{--}$ sector, we have both a $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ and a $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ with the same orbital angular momentum (both are F-waves) and the same isospin. The difference in their masses, which is around 250MeV, is a rough indication of the difference in binding energies of the $\mathcal{Q}_1$ and $\mathcal{Q}_2$ constituents. Therefore, the binding energy of the $\mathcal{Q}_2$ is roughly [7] 125MeV less than the binding energy of the $\mathcal{Q}_1$.[8]

This implies that the $\mathcal{Q}_2$ is lighter than the "bad" diquark $\mathcal{Q}_3$, which is believed to be about $200 - 300$MeV heavier than $\mathcal{Q}_1$ [12, 9].

3.7 Decays of diquark–antidiquark mesons and the $N\bar{N}$ threshold

Our model is schematic and thus it is not intended to provide detailed predictions about decays of the three types of mesons in our model.[9] However, we can still use our model to say something about these decays.

There is a clear distinction between the expected decays of $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ mesons and $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ mesons [38, 39, 40]. This distinction is due to the fact that $\mathcal{Q}_1$ is a color antitriplet ($\bar{\mathbf{3}}_c$),

[7]This rough estimate does not take into account the difference between binding of $\mathcal{Q}_1$ to $\bar{\mathcal{Q}}_1$ and the binding of $\mathcal{Q}_2$ to $\bar{\mathcal{Q}}_2$.

[8]This is consistent with the difference in their binding energies under the interaction $\mathcal{H}_{CM}$ ($\Delta E = (-8 + 4/3) \cdot 20\text{MeV} = 133\text{MeV}$); see Table 1 and Section 5.

[9]As pointed out in [34], the data for decay amplitudes and branching fractions for mesons is anyway far from accurate, making it difficult to test any strong decay models [35, 36, 37].

while $\mathcal{Q}_2$ is a color sextet ($\mathbf{6}_c$).

When a quark–antiquark pair is produced from the vacuum, the quark – which is a color triplet ($\mathbf{3}_c$) – can join the diquark $\mathcal{Q}_1$ to form a baryon, since their tensor product contains a color singlet:

$$q\mathcal{Q}_1 : \qquad \mathbf{3}_c \otimes \bar{\mathbf{3}}_c = \mathbf{8}_c \oplus \mathbf{1}_c \qquad SU(3)_c. \tag{17}$$

Similarly, the antiquark can join the antidiquark to form an antibaryon. When these processes are put together, the quark–antiquark pair joins the $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ to form a loosely bound baryon–antibaryon molecule which would dissociate quickly.

The $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ is protected from such a process since a color sextet cannot join a quark or antiquark to form the color singlet necessary for the formation of a baryon. This can be seen from the absence of $\mathbf{1}_c$ in the following decompositions:

$$q\mathcal{Q}_2 : \qquad \mathbf{3}_c \otimes \mathbf{6}_c = \mathbf{10}_c \oplus \mathbf{8}_c \qquad SU(3)_c, \tag{18}$$

$$\bar{q}\mathcal{Q}_2 : \qquad \bar{\mathbf{3}}_c \otimes \mathbf{6}_c = \mathbf{15}_c \oplus \mathbf{3}_c \qquad SU(3)_c. \tag{19}$$

Therefore, we would not expect to see $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ mesons above the nucleon–antinucleon threshold (around 2GeV for light mesons); if any such states do exist, they should be very broad and difficult to detect. On the other hand, $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ mesons above 2GeV may be narrow.

Our classification shows (Table 3a) that indeed, there are no light $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ above the nucleon–antinucleon threshold.

4 The Baryon Sector

We have stated that there are no radial excitations in the meson spectrum. Can we make an analogous statement about the baryon spectrum? As we show in this section, the answer is "yes." Note that we will not carry out a reclassification of the entire baryon spectrum since in essence this has already been done [9, 10, 41, 42].

4.1 Diquark building blocks for baryons

A baryon, like any hadron, must be a singlet under the color group. If we assume that a baryon is made of a quark and a diquark, then in order for a quark–diquark state to contain a color singlet corresponding to a baryon, the diquark has to be a color antitriplet $\bar{\mathbf{3}}_c$:

$$q\mathcal{Q}: \qquad \mathbf{3}_c \otimes \bar{\mathbf{3}}_c = \mathbf{8}_c \oplus \mathbf{1}_c \qquad SU(3)_c\,. \tag{20}$$

If the diquark were a color sextet $\mathbf{6}_c$, combining it with a quark would not result in a color singlet so no hadron could form:

$$q\mathcal{Q}: \qquad \mathbf{3}_c \otimes \mathbf{6}_c = \mathbf{10}_c \oplus \mathbf{8}_c \qquad SU(3)_c\,. \tag{21}$$

Therefore, the diquark building blocks for the baryon sector are the color–antisymmetric ones, $\mathcal{Q}_1$ and $\mathcal{Q}_3$ (see Table 1 and [9, 10, 41, 42]).

The flavor multiplets that can be obtained from $\mathcal{Q}_1$ are octets and singlets while from $\mathcal{Q}_3$ we obtain decuplets and octets:

$$q\mathcal{Q}_1 \;: \qquad \mathbf{3}_f \otimes \bar{\mathbf{3}}_f = \mathbf{8}_f \oplus \mathbf{1}_f \qquad SU(3)_f, \tag{22}$$

$$q\mathcal{Q}_3 \;: \qquad \mathbf{3}_f \otimes \mathbf{6}_f = \mathbf{10}_f \oplus \mathbf{8}_f \qquad SU(3)_f. \tag{23}$$

4.2 Baryons and radials

There are only a few baryons that have been believed to be candidates for radial excitations, as classified in the literature and the PDG (Table 14.6 of [8]). The lightest one, $N(1440)$, is known as the Roper resonance [43, 44]. The full list of light ones is:

$$\begin{aligned} &1/2^+: \; N(1440),\; \Lambda(1600),\; \Sigma(1660); \quad (\text{"Roper octet"}) \qquad (24)\\ &1/2^+: \; N(1710),\; \Lambda(1810),\; \Sigma(1880);\\ &3/2^+: \; \Delta(1600). \end{aligned}$$

It was shown in a different context in [7, 10] that these baryons can be identified with orbitally excited states of the form $\mathcal{Q}_1\mathcal{Q}_1\bar{q}$, where the two $\mathcal{Q}_1$ diquarks are in a relative P–wave. Specifically, the $N(1440)$, $\Lambda(1600)$, $\Sigma(1660)$, $N(1710)$, and a Σ around 1850MeV

are $\mathcal{Q}_1\mathcal{Q}_1\bar{q}$ states in an $SU(3)_f$ $\mathbf{8}_f \oplus \bar{\mathbf{10}}_f$ with $J^P = 1/2^+$ and $L = 1$ (L denotes the relative orbital angular momentum between the $\mathcal{Q}_1$'s) and no radial quantum number. Similarly, we suggest that the $\Delta(1600)$ is a $\mathcal{Q}_1\mathcal{Q}_1\bar{q}$ state belonging to an $SU(3)_f$ $\mathbf{8}_f \oplus \bar{\mathbf{10}}_f$ with $J^P = 3/2^+$ and $L = 1$, again with no radial quantum number.

So, just as in the meson spectrum, there are no radial excitations in the baryon spectrum either.

5 Interquark Forces in Mesons and Baryons

As we noted before, the interquark forces of low–energy QCD are not known, so we ended up deriving the diquark building blocks from meson and baryon phenomenology and properties of color and flavor representations. We found that the diquark building blocks of mesons are flavor-antisymmetric ($\mathcal{Q}_1$ and $\mathcal{Q}_2$ in Table 1), while the diquark building blocks of baryons are color-antisymmetric ($\mathcal{Q}_1$ and $\mathcal{Q}_3$ in Table 1). Since the diquarks in the meson sector are different from the diquarks in the baryon sector, the interquark forces in the meson and baryon sectors must also be different.

It is now natural to seek to learn something about the interquark interactions from these phenomena.

As it happens, there is an interaction under which the attractive diquark configurations are the flavor–antisymmetric ones, $\mathcal{Q}_1$ and $\mathcal{Q}_2$, as in the meson spectrum. That interaction is the spin–dependent part of one gluon exchange (OGE), also known as the color–magnetic interaction $\mathcal{H}_{CM}$. It was introduced as an important ingredient of hadron spectroscopy in [45], and it is given by

$$\mathcal{H}_{CM} \propto -\lambda_1 \cdot \lambda_2 \, \sigma_1 \cdot \sigma_2, \tag{25}$$

where σ_i and λ_i are respectively the spin and color operators of the ith quark (the spin–orbit interaction terms of OGE vanish for ground–state diquarks [45]).

The values of $\mathcal{H}_{CM}$ for each diquark configuration can be obtained by defining flavor, spin, and color exchange operators P_f, P_s, and P_c, which equal $+1$ if the quarks are symmetric under the corresponding exchanges, and -1 if they are antisymmetric [14]; then $\mathcal{H}_{CM}$ can

be rewritten as follows:

$$\mathcal{H}_{CM} \propto 4P_f + \frac{4}{3}P_s + 2P_c - \frac{2}{3} \ . \tag{26}$$

We can see that flavor exchange, P_f, plays the dominant role, as it has the largest coefficient. In effect, it makes a configuration attractive whenever it is antisymmetric in flavor. In Table 1, we have included $\mathcal{H}_{CM}$ for each diquark configuration (in units of about 20MeV; see [38, 14]), and we see that it is negative for $\mathcal{Q}_1$ and $\mathcal{Q}_2$, the diquark building blocks for mesons.

There is also an interaction under which the attractive diquark configurations are the color-antisymmetric ones, $\mathcal{Q}_1$ and $\mathcal{Q}_3$, as in the baryon spectrum. That interaction is the spin–independent part of OGE, also known as the color–electrostatic interaction. It is given by

$$\mathcal{H}_{CE} \propto \lambda_1 \cdot \lambda_2 = 2P_c - 2/3 \ . \tag{27}$$

As displayed in Table 1, the value of $\mathcal{H}_{CE}$ is $-8/3$ for the color antitriplet $\bar{\mathbf{3}}_c$ and $4/3$ for the color sextet $\mathbf{6}_c$ (again in units of 20MeV); both $\mathcal{Q}_1$ and $\mathcal{Q}_3$ are attractive under this interaction and form the diquark building blocks for baryons.

We deduce that in the meson sector, the interquark forces have qualitative similarities with the spin–dependent, color-magnetic part of OGE, and in the baryon sector the interquark forces are qualitatively similar to the spin–independent, color-electrostatic part of OGE. This distinction between the interquark forces in the meson and baryon sectors should be taken into account in the construction of any dynamical model for low–energy QCD.

6 Regge Trajectories of Mesons

Regge trajectories are families of hadrons which have the same internal spin and isospin and the same alignment of internal spin with orbital angular momentum. They are arranged in "trajectories" of increasing orbital angular momentum L. The squared masses of hadrons in a trajectory are expected to increase linearly with L [46]:

$$m^2 = a + \sigma_\alpha L \ , \tag{28}$$

Table 4a: Regge trajectories of light $q\bar{q}$

Table 4a (I): light $q\bar{q}$ mesons, S=1, S and L aligned								
L	S	J^{PC}	$[I=1]$	m^2 (GeV^2)	$[I=1/2]$	m^2 (GeV^2)	$[I=0]$	m^2 (GeV^2)
0	1	1^{--}	$\rho(770)$	0.6	$K(892)$	0.8	$\omega(782), \phi(1020)$	$0.6, 1.0$
1	1	2^{++}	$a_2(1320)$	1.7	$K_2^*(1430)$	2.0	$f_2(1270), f_2(1430)$	$1.6, 2.0$
2	1	3^{--}	$\rho_3(1690)$	2.8	$K_3(1780)$	3.2	$\omega_3(1670), \phi_3(1850)$	$2.8, 3.4$
3	1	4^{++}	$a_4(2040)$	4.2	$K_4^*(2045)$	4.2	$f_4(2050), f_4(2220)$	$4.2, 4.9$
$\sigma_{q\bar{q}}$				1.1		1.1		$1.2, 1.2$

Table 4a (II): light $q\bar{q}$ mesons, S=0								
L	S	J^{PC}	$[I=1]$	m^2 (GeV^2)	$[I=1/2]$	m^2 (GeV^2)	$[I=0]$	m^2 (GeV^2)
0	0	0^{-+}	$\pi(135)$	0.02	$K(494)$	0.2	$\eta(547), \eta'(958)$	$0.3, 0.9$
1	0	1^{+-}	$b_1(1235)$	1.5	$K_1(1270)$	1.6	$h_1(1170), h_1(1380)$	$1.4, 1.9$
2	0	2^{-+}	$\pi_2(1670)$	2.8	$K_2(1770)$	3.1	$\eta_2(1645), \eta_2(1870)$	$2.7, 3.5$
$\sigma_{q\bar{q}}$				1.3		1.5		$1.3, 1.3$

where m is the mass of the hadron, and a is an intercept that depends on the trajectory. The slope of the trajectory is σ_α, where α is an index denoting the type of hadron. Here, α may denote $q\bar{q}$, $\mathcal{Q}_1\bar{\mathcal{Q}}_1$, or $\mathcal{Q}_2\bar{\mathcal{Q}}_2$.

We list trajectories of light $q\bar{q}$, $\mathcal{Q}_1\bar{\mathcal{Q}}_1$, and $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ mesons in Table 4a, Table 4b, and Table 4c, respectively. We list trajectories for charmed mesons in Tables 4d and 4e, and for bottom mesons in Tables 4f and 4g. A rough approximation for the slopes σ_α, which for light mesons are of order 1GeV per unit of orbital angular momentum and for heavy mesons are much higher, appears in the final row of each table. For Regge trajectories of baryons, see [9, 10].

Table 4a: Regge trajectories of light $q\bar{q}$, continued

Table 4a (III): light $q\bar{q}$ mesons, S=1, S and L antialigned								
L	S	J^{PC}	$[I=1]$	m^2 (GeV^2)	$[I=1/2]$	m^2 (GeV^2)	$[I=0]$	m^2 (GeV^2)
1	1	0^{++}	$a_0(1450)$	2.1	$K_0^*(1430)$	2.0	$f_0(1370), f_0(1710)$	$1.9, 2.9$
2	1	1^{--}	$\rho(1700)$	2.9	$K(1680)$	2.8	$\omega(1650)$	2.7
3	1	2^{++}			$K_2^*(1980)$	4.0	$f_2(1950), f_2(2010)$	$3.8, 4.0$
$\sigma_{q\bar{q}}$				0.8		1.0		1.0,

Table 4a (IV): light $q\bar{q}$ mesons, S=1, S and L partially aligned								
L	S	J^{PC}	$[I=1]$	m^2 (GeV^2)	$[I=1/2]$	m^2 (GeV^2)	$[I=0]$	m^2 (GeV^2)
1	1	1^{++}	$a_1(1260)$	1.6	$K_1(1400)$	2.0	$f_1(1285), f_1(1420)$	$1.7, 2.0$
2	1	2^{--}			$K_2(1820)$	3.3		
$\sigma_{q\bar{q}}$						1.3		

Table 4b: Regge trajectories of light $\mathcal{Q}_1\bar{\mathcal{Q}}_1$

light $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ mesons, S=0								
L	S	J^{PC}	$[I=1]$	m^2 (GeV^2)	$[I=1/2]$	m^2 (GeV^2)	$[I=0]$	m^2 (GeV^2)
0	0	0^{++}	$a_0(980)$	0.8			$f_0(600), f_0(980)$	$0.4, 0.8$
1	0	1^{--}	$\rho(1450)$	2.1	$K^*(1410)$	2.0	$\omega(1420), \phi(1680)$	$2.0, 2.8$
2	0	2^{++}	$a_2(1700)$	2.9			$f_2(1640)$	2.7
3	0	3^{--}	$\rho_3(1990)$	4.0				
$\sigma_{\mathcal{Q}_1\bar{\mathcal{Q}}_1}$				1.0				$1.2, 2.0$

Table 4c: Regge trajectories of light $\mathcal{Q}_2\bar{\mathcal{Q}}_2$

light $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ mesons, S=1, S and L partially aligned								
L	S	J^{PC}	$[I=1]$	m^2 (GeV^2)	$[I=1/2]$	m^2 (GeV^2)	$[I=0]$	m^2 (GeV^2)
1	1	0^{-+}	$\pi(1300)$	1.7	$K(1460)$	2.1	$\eta(1295), \eta(1475)$	$1.7, 2.2$
2	1	1^{+-}					$h_1(1595)$	2.5
3	1	2^{-+}	$\pi_2(2100)$	4.4	$K_2(2250)$	5.1		
$\sigma_{\mathcal{Q}_2\bar{\mathcal{Q}}_2}$				1.4		1.5		0.8

Table 4d: Regge trajectories of charmed $q\bar{q}$

charmed $q\bar{q}$ mesons, S=1, S and L aligned								
L	S	J^{PC}	$[I=1/2]$	m^2 (GeV^2)	$[I=0]$	m^2 (GeV^2)	$[I=0]$	m^2 (GeV^2)
0	1	0^{++}	D^*	4.0	D_s^*	4.5	$J/\psi(1S)$	9.6
1	1	1^{--}	$D_2^*(2460)$	6.1	$D_{s2}(2573)$	6.6	$\chi_{c2}(1P)$	12.6
$\sigma^c_{q\bar{q}}$				2.1		2.1		3

Table 4e: Regge trajectories of charmed $\mathcal{Q}_1\bar{\mathcal{Q}}_1$

charmed $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ mesons, S=0								
L	S	J^{PC}	$[I=1/2]$	m^2 (GeV^2)	$[I=0]$	m^2 (GeV^2)	$[I=0]$	m^2 (GeV^2)
0	0	0^{++}	$D_0^*(2400)$	5.8	$D_{s0}^*(2317)$	5.4	$\chi_{c0}(1P)$	11.6
1	0	1^{--}					$\psi(2S)$	13.6
2	0	2^{++}					$\chi_{c2}(2P)$	15.4
$\sigma^c_{\mathcal{Q}_1\bar{\mathcal{Q}}_1}$								1.9

Table 4f: Regge trajectories of bottom $q\bar{q}$

bottom $q\bar{q}$ mesons, S=1, S and L aligned								
L	S	J^{PC}	$[I=1/2]$	m^2 (GeV^2)	$[I=0]$	m^2 (GeV^2)	$[I=0]$	m^2 (GeV^2)
0	1	0^{++}	B^*	28.4	B_s^*	29.3	$\Upsilon(1S)$	89.5
1	1	1^{--}	$B_2^*(5747)$	33.0	$B_{s2}^*(5840)$	34.1	$\chi_{b2}(1P)$	98.2
$\sigma_{q\bar{q}}^b$				4.6		4.8		8.7

Table 4g: Regge trajectories of bottom $\mathcal{Q}_1\bar{\mathcal{Q}}_1$

bottom $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ mesons, S=0								
L	S	J^{PC}	$[I=1/2]$	m^2 (GeV^2)	$[I=0]$	m^2 (GeV^2)	$[I=0]$	m^2 (GeV^2)
0	0	0^{++}					$\chi_{b0}(1P)$	97.2
1	0	1^{--}					$\Upsilon(2S)$	100.5
2	0	2^{++}					$\chi_{b2}(2P)$	105.5
$\sigma_{\mathcal{Q}_1\bar{\mathcal{Q}}_1}^b$								4.2

Acknowledgements

I am grateful to Frank Wilczek, who gave me a glimpse into his work on baryon systematics, and in response to my question ”what about mesons?” encouraged me to pursue them. This work is the result. I am also grateful to Robert L. Jaffe, Howard Georgi, Richard Brower, Usha Mallik, Hulya Guler, Dan Pirjol, and Ayana Holloway for helpful discussions. This work was supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement DE-FC02-94ER40818.

Appendix

A Meson Classification Details

In this appendix we provide a multiplet by multiplet discussion of the classification.

A.1 Light mesons

$\mathbf{J^{PC} = 0^{-+}}$

We have two 0^{-+} nonets. Available assignments are an S–wave of $q\bar{q}$ and a P–wave of $\mathcal{Q}_2\bar{\mathcal{Q}}_2$. The orbital excitation rule tells us to assign the lower-lying nonet to the S–wave and the second nonet to the P–wave. Other 0^{-+} are isorons.

We took the $\eta(1475)$ to be the heavier isosinglet in the second nonet, leaving out the $\eta(1405)$. Our choice is due to the fact that the heavier isosinglet in any nonet should couple to kaons, and the $\eta(1475)$ couples to kaons more strongly than $\eta(1405)$ (see “Note on $\eta(1405)$” in [8]).

Note that the second nonet was previously taken to consist of radially excited mesons [8].

$\mathbf{J^{PC} = 0^{++}}$

The lightest scalar nonet is $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ with $L = S = 0$; an assignment of these mesons to four–quark states was suggested by Jaffe in 1976 [11]; see also [47].

The next nonet is the quark model's $q\bar{q}$ P–wave. The choice of isoscalar that would complete this nonet has always been ambiguous [48, 49, 50]. Following [50], we choose the $f_0(1710)$. The other f_0 mesons are isorons.

The third (partial) nonet has masses around 2GeV, so by the orbital excitation rule it should be either a D–wave or an F–wave; the only option is a $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ D–wave.

$\mathbf{J^{PC} = 1^{--}}$

There are three complete or close-to-complete 1^{--} nonets, and two incomplete nonets which consist of only the isospin triplet (the ρ). Available assignments are ${}^3S_1$ or ${}^3D_1$ of $q\bar{q}$, ${}^1P_1$ of $\mathcal{Q}_1\bar{\mathcal{Q}}_1$, and ${}^1P_1$ or ${}^5P_1$ or ${}^5F_1$ of $\mathcal{Q}_2\bar{\mathcal{Q}}_2$. By the orbital excitation rule, the lowest-lying nonet, with masses less than 1GeV, is an S-wave so we assign it to ${}^3S_1$ of $q\bar{q}$.

The second nonet is about .5 GeV heavier, so it is a P-wave of either $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ or $\mathcal{Q}_2\bar{\mathcal{Q}}_2$. We assign it to ${}^1P_1$ of $\mathcal{Q}_1\bar{\mathcal{Q}}_1$, though this choice is rather arbitrary – this nonet could be a mixture of $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ and $\mathcal{Q}_2\bar{\mathcal{Q}}_2$.

The next nonet has only the $\rho(1570)$, which appeared in the PDG for the first time in 2008. It is slightly heavy for a P-wave by the orbital excitation rule, but we still assign it to the P-wave of $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ because there is a more suitable nonet for the available D-wave assignment; since it is heavy relative to other P-waves, we choose the ${}^5P_1$ rather than the ${}^1P_1$ assignment because it is plausible that higher S may mean higher mass (also see equation (26)).

The next nonet, which is about 1GeV higher than the lightest nonet, is a D–wave by the orbital excitation rule and we assign it to ${}^3D_1$ of $q\bar{q}$. Another isovector is at the mass range of F–waves, and we assign it to $\mathcal{Q}_2\bar{\mathcal{Q}}_2$.

Note that the second 1^{--} nonet was previously taken to consist of radially excited mesons [8].

$\mathbf{J^{PC} = 1^{++}}$

There are two nonets. Available assignments are a P–wave of $q\bar{q}$ and a D-wave of $\mathcal{Q}_2\bar{\mathcal{Q}}_2$. Using the orbital excitation rule, we assign the lighter nonet to a P–wave and the second nonet to a D–wave.

$\mathbf{J^{PC} = 1^{-+}}$

There are no complete nonets here. However, from Table 2 we know that a 1^{-+} nonet should appear as $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ in a P–wave. We classify the $\pi_1(1600)$ and $K(1630)$ as members of this nonet even though it is a bit heavy for a P–wave (we could have taken the $\pi_1(1400)$, but we opted to make the nonet consist of mesons whose masses are closer together); the $\pi_1(1400)$ is an isoron. Note that it has been argued [31, 51] that if the 1^{-+} pion is a four–quark state, then it should be part of a large flavor multiplet, i.e. larger than a nonet. Such a multiplet has not been observed, and in our model it is not expected to be - we expect only nonets in the light flavor sector (Section 2.1). See [8, 32, 52] for more on the 1^{-+} pions.

$\mathbf{J^{PC} = 2^{-+}}$

There are three nonets, and there are three available assignments: a ${}^3P_2$ of $\mathcal{Q}_2\bar{\mathcal{Q}}_2$, a ${}^1D_2$ of $q\bar{q}$, and a ${}^3F_2$ of $\mathcal{Q}_2\bar{\mathcal{Q}}_2$. We assign the lightest nonet to the P-wave (even though it is a bit heavy for a P-wave), the next one to the D-wave, and the last one to the F–wave. Note that the second nonet has so far only the isovector $\pi_2(1880)$, which in fact entered the PDG only in 2008; if it were not for its appearance, we would have assigned the lightest nonet to the D-wave based on the orbital excitation rule.

$\mathbf{J^{PC} = 2^{--}}$

There are two 2^- kaons here. We assign the lighter to a P-wave (though it's a bit heavy based on the orbital excitation rule) and the heavier to a D-wave.

$\mathbf{J^{PC} = 2^{++}}$

There are three almost complete nonets. The lightest and heaviest are both $q\bar{q}$, while the middle one is a $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ in a D–wave. The other two have only a single isoscalar in each; they are $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ D-waves. The $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ isoscalars and the isoscalars in the middle nonet, all D-waves, could mix.

$\mathbf{J^{PC} = 3^{--}}$

There is one complete nonet, which is the D–wave of $q\bar{q}$. There are also two heavier isovectors with the same J^{PC}. Of those, the lighter one is below the baryon–antibaryon threshold, so may be $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ in an F–wave. The second is above this threshold and therefore is unlikely to have $\mathcal{Q}_1$ as a constituent (see decay properties, p. 17); therefore, we assign it to $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ as an F–wave.

$\mathbf{J^{PC} = 4^{++}}$

There is one complete nonet in this sector, a $q\bar{q}$ in an F–wave. An $f_4(2300)$ should be an F-wave by the orbital excitation rule, but there are no available assignments, so it is an isoron.

$\mathbf{J^{PC} = 5^{--}}$

The 5^{--} nonet could be a G-wave $q\bar{q}$ or an F–wave $\mathcal{Q}_2\bar{\mathcal{Q}}_2$, or a $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ with even higher L. By the orbital excitation rule, it should be an F–wave, so we assign it to $\mathcal{Q}_2\bar{\mathcal{Q}}_2$. However, it could be a G–wave as classified in the PDG.

$\mathbf{J^{PC} = 6^{++}}$

The 6^{++} has the mass range appropriate for an F–wave or at most a G–wave. The lowest L available for this J^{PC} is a G–wave of $\mathcal{Q}_2\bar{\mathcal{Q}}_2$, which is our assignment. However, it could also be the H–wave as classified in the PDG.

A.2 Charmed and bottom mesons

$\mathbf{J^{PC} = 0^{-+}}$

There is one complete multiplet and one partial multiplet. Note that the J^{PC} of the bottom mesons in the first multiplet have not been determined experimentally. As is standard, we assign them to S–wave of $q\bar{q}$.

$\mathbf{J^{PC} = 0^{++}}$

Recent suggestions (see [54] for reviews) that $D^*_{s0}(2317)$ may be a tetraquark support the possibility that it completes the $\mathcal{Q}_1\bar{\mathcal{Q}}_1$ nonet rather than the $q\bar{q}$ nonet.

$\mathbf{J^{PC} = 1^{--}}$

Note that since its first appearance in the 1970's, the $\psi(2S)$ was assigned to be a radial excitation [55]. Until today, this assignment does not seem to have ever been questioned or challenged and is even part of the particle's name. In our paper, the $\psi(2S)$ is a diquark-antidiquark P–wave ($L = 1$).

$\mathbf{J^{PC} = 1^{++}}$

There are two multiplets, one complete and one incomplete. Recent suggestions (see [54] for reviews) that $D^*_{s1}(2460)$ may be a tetraquark support our classification to $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ rather than $q\bar{q}$.

We classify the $X(3872)$ as a member of the $\mathcal{Q}_2\bar{\mathcal{Q}}_2$ as well. The $J^{PC} = 1^{++}$ assignment for this particle is favored [56] but 2^{-+} is also possible [57]; see also [58]. Its isospin has not been determined yet; we list it under $I = 0$, but it may also be $I = 1$.

Note that we include the new bottom mesons B_1 and B_{s1}; Table 14.3 of the PDG does not include them.

$\mathbf{J^{PC} = 2^{++}}$

There is one complete multiplet and one partial one. We include the new B_2^* and B_{s2}^* (which do not appear in Table 14.3 of the PDG).

B "Exotic" and Outcast Mesons

We list in Table 5 all the mesons that appear in the 2008 PDG particle listing but are not classified in Tables 14.2 and 14.3 there. We do not include any of the mesons listed under "further states" in the PDG (those have not been confirmed).

Table 5: "Exotic" and Outcast Mesons in the 2008 PDG

J^{PC}	
0^{-+}	$\bullet\pi(1800), \bullet\eta(1405), \eta(1760), \eta(2225)$
0^{++}	$\bullet a_0(980), \bullet f_0(600), \bullet f_0(980), \bullet, f_0(1500),$
	$f_0(2020), f_0(2100), f_0(2200), f_0(2330)$
1^{--}	$\rho(1570), \rho(1900), \rho(2150), \phi(2170), \bullet\psi(4040),$
	$\bullet\psi(4160), Y(4260), \bullet X(4260), X(4360) \bullet \psi(4415),$
	$X(4660), \bullet\Upsilon(3S), \bullet\Upsilon(4S), \bullet\Upsilon(10860), \bullet\Upsilon(11020)$
1^{-+}	$\bullet\pi_1(1400), \bullet\pi_1(1600),$
1^{++}	$a_1(1640), K_1(1650), f_1(1510)$
1^{+-}	$h_1(1595)$
2^{-+}	$\bullet\pi_2(1880), \pi_2(2100),$
2^{--}	$\Upsilon(1D)$
2^{++}	$f_2(1430), f_2(1565), f_2(1640), a_2(1700), f_2(1810),$
	$f_2(1910), \bullet f_2(1950), \bullet f_2(2010), f_2(2150), f_J(2220),$
	$\bullet f_2(2300), \bullet f_2(2340), \chi_{c2}(2P)$
3^{--}	$\rho_3(1990), \rho_3(2250)$
4^{++}	$f_4(2300)$

J^P	
0^-	$K(1830)$
0^+	$\kappa(800), K_0^*(1950),$
1^+	$K_1(1650), \bullet B_1(5721)^0, \bullet B_{s1}(5830)^0$
2^-	$K_2(1580), K_2(2250)$
2^+	$K_2^*(1980), B_2^*(5747)^0, B_{s2}^*(5840)^0$
3^+	$K_3(2320)$
4^-	$K_4(2500)$
5^-	$K_5^*(2380)$
$?^?$	$K(1630), K(3100), D^*(2640),$
	$Y(3940), B_J^*(5732), B_{sJ}^*(5850),$
	$h_c(1P)$

References

[1] T. Friedmann, ”No radial excitations in low energy QCD. II. The shrinking radius of hadrons,” arXiv:0910.2231.

[2] Y. Ne’eman, “Derivation of strong interactions from a gauge invariance,” Nucl. Phys. **26**, 222 (1961).

M. Gell-Mann, ”The Eightfold Way: A theory of strong interaction symmetry,” California Institute of Technology Synchrotron Laboratory Report CTSL-20 (1961), unpublished.

M. Gell-Mann, “A schematic model of baryons and mesons,” Phys. Lett. **8**, 214 (1964).

G. Zweig, CERN preprint 8419/Th412, 8182/Th401 (unpublished).

M. Gell-Mann and Y. Ne’eman, ”The Eightfold Way,” (New York: W. A. Benjamin, Inc., 1964).

[3] M. Anselmino, E. Predazzi, S. Ekelin, S. Fredriksson and D. B. Lichtenberg, “Diquarks,” Rev. Mod. Phys. **65**, 1199 (1993).

[4] C. Amsler and N. A. Tornqvist, “Mesons beyond the naive quark model,” Phys. Rept. **389**, 61 (2004).

[5] T. Nakano *et al.* [LEPS Collaboration], “Observation of S = +1 baryon resonance in photo-production from neutron,” Phys. Rev. Lett. **91**, 012002 (2003) [arXiv:hep-ex/0301020].

[6] R. L. Jaffe, “Exotica,” Phys. Rept. **409**, 1 (2005) [Nucl. Phys. Proc. Suppl. **142**, 343 (2005)] [arXiv:hep-ph/0409065].

[7] R. L. Jaffe and F. Wilczek, “Diquarks and exotic spectroscopy,” Phys. Rev. Lett. **91**, 232003 (2003) [arXiv:hep-ph/0307341].

[8] C. Amsler *et al.* [Particle Data Group], “Review of particle physics,” Phys. Lett. B **667**, 1 (2008); W. M. Yao *et al.* [Particle Data Group], “Review of particle physics,” J. Phys. G **33**, 1 (2006).

[9] F. Wilczek, “Diquarks as inspiration and as objects,” arXiv:hep-ph/0409168.

[10] A. Selem and F. Wilczek, “Hadron systematics and emergent diquarks,” arXiv:hep-ph/0602128.

[11] R. L. Jaffe, “Multi - quark hadrons. 1. The phenomenology of $q^2\bar{q}^2$ mesons,” Phys. Rev. D **15**, 267 (1977).

[12] C. Rosenzweig, “Have mesons composed of charmed diquarks been discovered?,” Phys. Rev. Lett. **36**, 697 (1976).

[13] H. J. Lipkin, "Are there charmed - strange exotic mesons?," Phys. Lett. B **70**, 113 (1977).

A. De Rujula, H. Georgi and S. L. Glashow, "Molecular charmonium: A new spectroscopy?," Phys. Rev. Lett. **38**, 317 (1977).

T. E. Browder, S. Pakvasa and A. A. Petrov, "Comment on the new $D_s^{(*)+}\pi^0$ resonances," Phys. Lett. B **578**, 365 (2004) [arXiv:hep-ph/0307054].

T. Barnes, F. E. Close and H. J. Lipkin, "Implications of a D K molecule at 2.32-GeV," Phys. Rev. D **68**, 054006 (2003) [arXiv:hep-ph/0305025].

H. Y. Cheng and W. S. Hou, "B decays as spectroscope for charmed four-quark states," Phys. Lett. B **566**, 193 (2003) [arXiv:hep-ph/0305038].

K. Terasaki, "BABAR resonance as a new window of hadron physics," Phys. Rev. D **68**, 011501 (2003) [arXiv:hep-ph/0305213].

B. Nicolescu and J. P. B. de Melo, "Is the $D_{sJ}(2632)^+$ meson a cryptoexotic tetraquark baryonium state?," arXiv:hep-ph/0407088.

K. Terasaki and B. H. J. McKellar, "Charmed scalar resonances: Conventional and four-quark mesons," Prog. Theor. Phys. **114**, 205 (2005) [arXiv:hep-ph/0501188].

L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, "Diquark-antidiquarks with hidden or open charm and the nature of X(3872)," Phys. Rev. D **71**, 014028 (2005) [arXiv:hep-ph/0412098].

S. Godfrey and S. L. Olsen, "The exotic XYZ charmonium-like mesons," arXiv:0801.3867 [hep-ph].

S. L. Olsen, "Hadronic spectrum - multiquark states," arXiv:0901.2371 [hep-ex].

H. Guler, "What are the $X(3872)$ and D_{sJ} particles?," *Prepared for Heavy Quarks and Leptons Workshop 2004, San Juan, Puerto Rico, 1-5 Jun 2004.*

T. Barnes, "Recent developments in charm spectroscopy: $X(3872)$, $D_{sJ}^*(2317)^+$ and $D_{sJ}(2463)^+$," *Prepared for Heavy Quarks and Leptons Workshop 2004, San Juan, Puerto Rico, 1-5 Jun 2004.*

E. Eichten *et al*, with updates by S. Eidelman, H. Mahlke-Kruger, C. Patrignani, "New charmonium-like states," in [8].

[14] R. L. Jaffe, "Color, spin, and flavor-dependent forces in quantum chromodynamics," arXiv:hep-ph/0001123.

[15] T. Burns, F. E. Close and J. J. Dudek, "Pentaquark implications for exotic mesons," Phys. Rev. D **71**, 014017 (2005) [arXiv:hep-ph/0411160].

[16] N. Isgur, "Critique of a pion exchange model for interquark forces," Phys. Rev. D **62**, 054026 (2000) [arXiv:nucl-th/9908028].

[17] M. G. Mayer, "On closed shells in nuclei," Phys. Rev. **74**, 235 (1948).

[18] C. J. Morningstar and M. J. Peardon, "The glueball spectrum from an anisotropic lattice study," Phys. Rev. D **60**, 034509 (1999) [arXiv:hep-lat/9901004].

[19] Y. Chen *et al.*, "Glueball spectrum and matrix elements on anisotropic lattices," Phys. Rev. D **73**, 014516 (2006) [arXiv:hep-lat/0510074].

[20] W. Lucha, F. F. Schoberl and D. Gromes, "Bound states of quarks," Phys. Rept. **200**, 127 (1991).

[21] S. J. Brodsky and R. Shrock, "Maximum wavelength of confined quarks and gluons and properties of quantum chromodynamics," Phys. Lett. B **666**, 95 (2008) [arXiv:0806.1535 [hep-th]].

G. F. de Teramond and S. J. Brodsky, "Light-front holography: a first approximation to QCD," Phys. Rev. Lett. **102**, 081601 (2009) [arXiv:0809.4899 [hep-ph]].

[22] N. Barash-Schmidt *et al.*, "Review of particle properties." Rev. Mod. Phys. **41**,109 (1969).

V. Chaloupka *et al.*, "Review of particle properties. Particle Data Group," Phys. Lett. B **50**, 1 (1974).

T. G. Trippe *et al.*, "Review of particle properties. Particle Data Group," Rev. Mod. Phys. **48**, S1 (1976) [Erratum-ibid. **48**, 497 (1976)].

C. Bricman *et al.*, "Review of particle properties. Particle Data Group," Phys. Lett. B **75**, 1 (1978).

[23] R. L. Jaffe, "Quark confinement," Nature **268**, 201 (1977).

[24] R. L. Kelly *et al.*, "Review of particle properties. Particle Data Group," Rev. Mod. Phys. **52**, S1 (1980).

[25] K. Hikasa *et al.* [Particle Data Group], "Review of particle properties. Particle Data Group," Phys. Rev. D **45**, S1 (1992) [Erratum-ibid. D **46**, 5210 (1992)].

[26] K. Hagiwara *et al.* [Particle Data Group], "Review of particle physics," Phys. Rev. D **66**, 010001 (2002).

[27] S. Eidelman *et al.* [Particle Data Group], "Review of particle properties," Phys. Lett. B **592**, 1 (2004).

[28] C. Amsler, private communication.

[29] J. L. Rosner, private communication.

[30] T. Barnes, "Radial excitations," arXiv:hep-ph/9907259.

A. M. Badalian, B. L. G. Bakker and Y. A. Simonov, "Light meson radial Regge trajectories," Phys. Rev. D **66**, 034026 (2002) [arXiv:hep-ph/0204088].

A. V. Anisovich *et al.*, "A $J^{PC} = 0^{-+}$ enhancement at the $f_0(980)\eta$ threshold," Phys. Lett. B **472**, 168 (2000).

T. Barnes, "Light meson spectroscopy: Recent developments and DAPHNE," arXiv:hep-ph/0001326.

T. Barnes, N. Black and P. R. Page, "Strong decays of strange quarkonia," Phys. Rev. D **68**, 054014 (2003) [arXiv:nucl-th/0208072].

D. Melikhov and O. Pene, "Is the resonance D(2637) really a radial excitation?," Phys. Lett. B **446**, 336 (1999) [arXiv:hep-ph/9809308].

[31] S. U. Chung, E. Klempt and J. G. Korner, "SU(3) classification of p-wave $\eta\pi$ and $\eta'\pi$ systems," Eur. Phys. J. A **15**, 539 (2002) [arXiv:hep-ph/0211100].

[32] D. R. Thompson *et al.* [E852 Collaboration], "Evidence for exotic meson production in the reaction $\pi^- p \to \eta\pi^- p$ at 18-GeV/c," Phys. Rev. Lett. **79**, 1630 (1997) [arXiv:hep-ex/9705011].

[33] D. Black, A. H. Fariborz, F. Sannino and J. Schechter, "Putative light scalar nonet," Phys. Rev. D **59**, 074026 (1999) [arXiv:hep-ph/9808415].

[34] T. Barnes, "Light meson spectroscopy: Recent developments and DAPHNE," arXiv:hep-ph/0001326.

[35] T. Barnes, N. Black and P. R. Page, "Strong decays of strange quarkonia," Phys. Rev. D **68**, 054014 (2003) [arXiv:nucl-th/0208072].

T. Barnes, F. E. Close, P. R. Page and E. S. Swanson, "Higher quarkonia," Phys. Rev. D **55**, 4157 (1997) [arXiv:hep-ph/9609339].

F. E. Close and P. R. Page, "How to distinguish hybrids from radial quarkonia," Phys. Rev. D **56**, 1584 (1997) [arXiv:hep-ph/9701425].

[36] R. Kokoski and N. Isgur, "Meson decays by flux tube breaking," Phys. Rev. D **35**, 907 (1987).

[37] E. S. Ackleh, T. Barnes and E. S. Swanson, "On the mechanism of open-flavor strong decays," Phys. Rev. D **54**, 6811 (1996) [arXiv:hep-ph/9604355].

[38] H. M. Chan and H. Hogaasen, "Strings, bags, and baryoniums," Phys. Lett. B **72**, 400 (1978); "Baryonium states in multiquark spectroscopy," Phys. Lett. B **72**, 121 (1977).

[39] R. L. Jaffe, "$Q^2\,\bar{Q}^2$ resonances in the baryon - anti-baryon system," Phys. Rev. D **17**, 1444 (1978).

[40] A. W. Hendry and I. Hinchliffe, “Metastable exotic mesons,” Phys. Rev. D **18**, 3453 (1978).

[41] M. Ida and R. Kobayashi, “Baryon resonances in a quark model,” Prog. Theor. Phys. **36**, 846 (1966).

[42] D. B. Lichtenberg and L. J. Tassie, ”Baryon mass splitting in a boson-fermion model,” Phys. Rev., **155**, 1601 (1967).

[43] T. A. DeGrand and C. Rebbi, “Radial excitations of hadronic bags,” Phys. Rev. D **17**, 2358 (1978).

[44] M. Rosina and B. Golli, “The Roper resonance revisited,” FizikaB **13**, 217 (2004).

[45] A. De Rujula, H. Georgi and S. L. Glashow, “Hadron masses in a gauge theory,” Phys. Rev. D **12**, 147 (1975).

[46] G. F. Chew and S. C. Frautschi, ”Regge trajectories and the principle of maximum strength for strong interactions,” Phys. Rev. Lett. **8**, 41 (1962).

[47] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, “A new look at scalar mesons,” Phys. Rev. Lett. **93**, 212002 (2004) [arXiv:hep-ph/0407017].

[48] C. Amsler,“Non–$q\bar{q}$ mesons,” in [8].

[49] S. Spanier and N. A. Tornqvist, ”Note on scalar mesons,” in [8].

[50] C. Amsler, “Further evidence for a large glue component in the $f_0(1500)$ meson,” Phys. Lett. B **541**, 22 (2002) [arXiv:hep-ph/0206104].

[51] F. E. Close, “The end of the constituent quark model?,” AIP Conf. Proc. **717**, 919 (2004) [arXiv:hep-ph/0311087].

[52] C. W. Bernard *et al.* [MILC Collaboration], “Exotic mesons in quenched lattice QCD,” Phys. Rev. D **56**, 7039 (1997) [arXiv:hep-lat/9707008].

T. Barnes, F. E. Close, F. de Viron and J. Weyers, “Q anti-q G hermaphrodite mesons in the MIT bag model,” Nucl. Phys. B **224**, 241 (1983).

P. Lacock, C. Michael, P. Boyle and P. Rowland [UKQCD Collaboration], “Hybrid mesons from quenched QCD,” Phys. Lett. B **401**, 308 (1997) [arXiv:hep-lat/9611011].

[53] L. Maiani, F. Piccinini, A. D. Polosa and V. Riquer, “Positive parity scalar mesons in the 1-GeV - 2-GeV mass range,” arXiv:hep-ph/0604018.

[54] H. Guler, “What are the $X(3872)$ and D_{sJ} particles?,” *Prepared for Heavy Quarks and Leptons Workshop 2004, San Juan, Puerto Rico, 1-5 Jun 2004.*

T. Barnes, “Recent developments in charm spectroscopy: $X(3872)$, $D^*_{sJ}(2317)^+$ and $D_{sJ}(2463)^+$,” *Prepared for Heavy Quarks and Leptons Workshop 2004, San Juan, Puerto Rico, 1-5 Jun 2004.*

[55] C. G. . Callan, R. L. Kingsley, S. B. Treiman, F. Wilczek and A. Zee, “Remarks on the new resonances at 3.1-Gev and 3.7-Gev,” Phys. Rev. Lett. **34**, 52 (1975).

B. J. Harrington, S. Y. Park and A. Yildiz, “The spectra of heavy mesons in e^+e^- annihilation,” Phys. Rev. Lett. **34**, 168 (1975) [Erratum-ibid. **34**, 296 (1975)].

T. Appelquist, A. De Rujula, H. D. Politzer and S. L. Glashow, “Charmonium spectroscopy,” Phys. Rev. Lett. **34**, 365 (1975).

E. Eichten, K. Gottfried, T. Kinoshita, J. B. Kogut, K. D. Lane and T. M. Yan, “The spectrum of charmonium,” Phys. Rev. Lett. **34**, 369 (1975) [Erratum-ibid. **36**, 1276 (1976)].

[56] K. Abe *et al.*, “Experimental constraints on the possible J^{PC} quantum numbers of the $X(3872)$,” arXiv:hep-ex/0505038.

[57] A. Abulencia *et al.* [CDF Collaboration], “Analysis of the quantum numbers J^{PC} of the $X(3872)$,” Phys. Rev. Lett. **98**, 132002 (2007) [arXiv:hep-ex/0612053].

[58] E. Eichten *et al*, with updates by S. Eidelman, H. Mahlke-Kruger, C. Patrignani, ”New charmonium-like states,” in [8].